Question			Answer	Marks	Guidance
1	(i)		$P(X \le x) = F_B(x).\frac{1}{2} + F_G(x).\frac{1}{2}$	M1	use of cdfs
			ie cdf of X is $F(x) = \frac{1}{2}\{F_B(x) + F_G(x)\}$	A1	
			ie (by differentiating) pdf of X is $f(x) = \frac{1}{2}\{f_B(x) + f_G(x)\}$	A1	Answer given
				[3]	
1	(ii)		$E(X)\left(= \frac{1}{2}\left\{\int x f_B(x)dx + \int x f_G(x)dx\right\}\right) = \frac{1}{2}\mu_B + \frac{1}{2}\mu_G$	M1	[answer given; needs *some* indication of method]
				[1]	
1	(iii)		$E(X^2) = \int x^2 f(x)dx$	M1	
			$= \frac{1}{2}\left\{\int x^2 f_B(x)dx + \int x^2 f_G(x)dx\right\}$	M1	
			Use of "$E(X^2) = \sigma^2 + \mu^2$"	M1	
			$= \frac{1}{2}\{\sigma^2 + \mu_B{}^2 \quad + \quad \sigma^2 + \mu_G{}^2\}$	A1	
			$\therefore \text{Var}(X) = E(X^2) - \{E(X)\}^2$	M1	
			$= \sigma^2 + \frac{1}{2}\mu_B{}^2 + \frac{1}{2}\mu_G{}^2 - \frac{1}{4}\mu_B{}^2 - \frac{1}{4}\mu_G{}^2 - \frac{1}{2}\mu_B\mu_G$	A1	
			$= \sigma^2 + \frac{1}{4}(\mu_B - \mu_G)^2$	A1	Answer given
				[7]	
1	(iv)		[Central Limit Theorem] Approx dist of $\bar{X}$ is		
			$N\left(\frac{1}{2}\mu_B + \frac{1}{2}\mu_G,\ \frac{1}{2n}\left(\sigma^2 + \frac{1}{4}(\mu_B - \mu_G)^2\right)\right)$		
			B1 B1 B1 B1	B4	4 marks as shown
				[4]	
1	(v)		$\bar{X}_{st} = \frac{1}{2}(\bar{X}_B + \bar{X}_G)$ $\quad \text{Var}(\bar{X}_{either}) = \frac{\sigma^2}{n}$	M1M1	
			$\therefore E(\bar{X}_{st}) = \frac{1}{2}(\mu_B + \mu_G)$	B1	
			and $\text{Var}(\bar{X}_{st}) = \frac{1}{4}\left(\frac{\sigma^2}{n} + \frac{\sigma^2}{n}\right) = \frac{\sigma^2}{2n}$	B1	
				[4]	

Question			Answer	Marks	Guidance
1	(vi)		$E(\bar{X}) = E(\bar{X}_{st}) = \frac{1}{2}(\mu_B + \mu_G) = E(X)$	E1	
			ie they are unbiased.	E1	
			Clearly $\text{Var}\left(\bar{X}\right) > \text{Var}\left(\bar{X}_{st}\right)$,	M1	for any attempt to compare variances Candidates are not required to note that the variances are equal in the case $\mu_B = \mu_G$.
				M1	for deduction that $\text{Var}\left(\bar{X}\right) > \text{Var}\left(\bar{X}_{st}\right)$ [FT c's variances]
			$\therefore$ $\bar{X}_{st}$ is the more efficient.	E1	More efficient
				[5]	
2	(i)		Mean of X = 3.5 (immediate by symmetry)	B1	
			$\text{E}\left(X^2\right) = \frac{1}{6}\left(1+4+...+36\right) = \frac{91}{6}$	M1	
			$\therefore \text{Var}\left(X\right) = \frac{91}{6} - \left(\frac{7}{2}\right)^2 = \frac{35}{12}$	A1	Answer given
				[3]	
2	(ii)		$\text{G}\left(t\right) = \text{E}\left(t^X\right) = \left(t^1.\frac{1}{6}\right) + \left(t^2.\frac{1}{6}\right) + ... + \left(t^6.\frac{1}{6}\right)$	M1	
			$= \frac{1}{6}\left(t + t^2 + ... + t^6\right) = \frac{t\left(1-t^6\right)}{6\left(1-t\right)}$	A1	Answer given
				[2]	
2	(iii)		[P(N = 0) = ½, P(N = 1) = (½)(½),] P(N = r) = $(½)^r$.(½)	B1 **[1]**	answer given; must be convincing
2	(iv)		$\text{H}\left(t\right) = \text{E}\left(t^N\right) = \left(t^0.\frac{1}{2}\right) + \left(t^1.\frac{1}{4}\right) + \left(t^2.\frac{1}{8}\right) + ...$	M1	
			$= \frac{\frac{1}{2}}{1-\frac{t}{2}} = \frac{1}{2-t} = \left(2-t\right)^{-1}$	A1	Answer given

Question			Answer	Marks	Guidance
2	(v)		Mean = H'(1), variance = H"(1) + mean – mean^2. $\mathrm{H}'(t)=(-1)(2-t)^{-2}(-1)=(2-t)^{-2}$ $\therefore$ mean = 1 $\mathrm{H}''(t)=(-2)(2-t)^{-3}(-1)=2(2-t)^{-3}$ $\therefore$ variance = 2 + 1 – 1 = 2	M1 A1 M1 A1 **[4]**	for <u>use</u> of 1st derivative for <u>use</u> of 2nd derivative For variance
2	(vi)		K(t) = H{G(t)} = {2 – G(t)}$^{-1}$ $=\left(2-\frac{t(1-t^6)}{6(1-t)}\right)^{-1}=\left(\frac{12(1-t)-t(1-t)(1+t+t^2+\ldots+t^5)}{6(1-t)}\right)^{-1}$ $=\left(\frac{12-t-t^2-t^3-\ldots-t^6}{6}\right)^{-1}=6(12-t-t^2-\ldots-t^6)^{-1}$	M1 M1 M1 A1 **[4]**	 inserting G(t) use of hint given Answer given
2	(vii)		$\mathrm{K}'(t)=6(12-t-t^2-\ldots-t^6)^{-2}(1+2t+3t^2+4t^3+5t^4+6t^5)$ $\mathrm{K}''(t)=12(12-t-t^2-\ldots-t^6)^{-3}(1+2t+3t^2+4t^3+5t^4+6t^5)^2$ $+\ 6(12-t-t^2-\ldots-t^6)^{-2}(2+6t+12t^2+20t^3+30t^4)$ $\therefore$ mean = K'(1) = 6(12 – 6)$^{-2}$(21) = 21/6 = 7/2 $\therefore$ K"(1) = (12 × 6^{-3} × 21^2) + (6 × 6^{-2} × 70) = (49/2) + (70/6) $\therefore$ variance = $\frac{49}{2}+\frac{70}{6}+\frac{7}{2}-\frac{49}{4}=\frac{294+140+42-147}{12}=\frac{329}{12}$	M1 M1 M1 A1 A1 A1 **[6]**	reasonable attempt to differentiate K(t) reasonable attempt at 2nd derivative for <u>use</u> of derivatives Substitution shown Ft c's K'(1) and/or K''(1) provided variance positive Exact.

Question			Answer	Marks	Guidance
2	(viii)		We have: $\mu_X = 7/2$ $\sigma_X{}^2 = 35/12$ $\mu_N = 1$ $\sigma_N{}^2 = 2$ $\sigma_Q{}^2 = 329/12$	M1	for correct use of candidate's values for means and variances
			Inserting in the quoted formula gives $\left[2\times\left(\frac{7}{2}\right)^2\right]+\left[1\times\frac{35}{12}\right] = \frac{294+35}{12} = \frac{329}{12}$ as required.	A1 **[2]**	answer honestly obtained (common denominator shown). A0 if different from (vii)
3	(i)		H_0: population medians are equal	B1	[*Note: "population" must be explicit*]
			H_1: population median for A < population median for B	B1	1) Explicit statement re shapes of distributions. (eg that they are the same shape) is not required. 2) More formal statements of hypotheses gain both marks [eg cdfs are F(x) and F($x - \Delta$), H_0 is $\Delta = 0$ etc).]
			Wilcoxon rank sum test (or Mann-Whitney form of test) Ranks are: A 1 2 4 5 9 11	M1	Combined ranking
			B 3 6 7 8 10 12 13 14	A1	Correct [allow up to 2 errors; FT provided M1 earned]
			W = 1 + 2 + 4 + 5 + 9 + 11 = 32 [or 0 + 0 + 1 + 1 + 4 + 5 = 11 if M-W used]	B1	
			Refer to $W_{6,8}$ [or $MW_{6,8}$] tables	M1	No FT if wrong
			Lower 5% critical point is 31 [or 10 if M-W used]	A1	No FT if wrong
			Result is not significant	A1	
			Seems median yields may be assumed equal	A1 **[9]**	

Question			Answer	Marks	Guidance
3	(ii)		H_0: population means are equal	B1	
			H_1: population mean for A < population mean for B	B1	*"population" must be explicit, either in words or by notation*
			For A: $\bar{x} = 11.4$, $s_{n-1}^2 = 1.912$ [$s_{n-1} = 1.38275$]	B1	For all. Use of s_n scores B0
			For B: $\bar{y} = 12.575$, $s_{n-1}^2 = 1.051$ [$s_{n-1} = 1.025$]		
			Pooled $s^2 = \dfrac{(5\times1.912)+(7\times1.051)}{12} = \dfrac{16.915}{12} = 1.4096$	M1	for any reasonable attempt at pooling (but *not* if s_n^2 used)
				A1	If correct
			Test statistic $= \dfrac{11.4-12.575}{\sqrt{1.4096}\sqrt{\frac{1}{6}+\frac{1}{8}}} = \dfrac{-1.175}{0.6412} = -1.83(25)$	M1 A1	Ft if incorrect
			Refer to t_{12}	M1	No FT if wrong
			Lower single-tailed 5% critical point is –1.782	A1	No FT if wrong *must compare –1.83 with –1.782 unless it is clear and explicit that absolute values are being used*
			Significant	A1	
			Seems mean yield for A is less than that for B	A1 **[11]**	
3	(iii)		t test is "more sensitive" if Normality is correct. Non-rejection of Normality supports t. But Wilcoxon is more reliable if not Normal – and we do not have *proof* of Normality.	E1 E1 E1 E1 **[4]**	
4	(i)		Latin square	B1	
			4 × 4 layout, with rows clearly representing casting techniques and columns clearly representing chemical compositions [*or vice versa*]	B2,1,0	B2 if completely correct, B1 if one point omitted
			Labels each appearing exactly once in each row and in each column	B1	for correct structure
			representing manufacturers.	B1 **[5]**	within the square

Question			Answer	Marks	Guidance
4	(ii)		Totals are 440.6 453.5 458.2 459.4 all from samples of size 4 Grand total 1811.7 "Correction factor" CF = $1811.7^2/16$ = 205141.06 Total SS = 205202.57 – CF = 61.5144 Between manufacturers SS = $\dfrac{440.6^2}{4}+\dfrac{453.5^2}{4}+\dfrac{458.2^2}{4}+\dfrac{459.4^2}{4}-\text{CF}$ = 205196.55 – CF = 55.4969 Residual SS (by subtraction) = 61.5144 – 55.4969 = 6.0175	M1 M1 A1	for attempt to form three sums of squares. for correct method for any two if each calculated SS is correct.
			Source of variation / SS / df / MS / MS ratio Between treatments / 55.4969 / 3 / 18.4989 / 36.89 Residual / 6.0175 / 12 / 0.5014(6) Total / 61.5144 / 15	B1 B1 M1 M1 A1	Between treatments df Residual df Method for calculating either mean square MS ratio cao at least 3sf, condone up to 6 sf
			Refer MS ratio to $F_{3,12}$. quotation of an extreme point (not just the 5%point); eg 1% point, 5.95, or 0.1% point, 10.80. Must be some reference to *very highly* significant and *very strong/overwhelming* evidence that "the manufacturers are not all the same".	M1 A1 A1 A1 **[12]**	No FT if wrong
4	(iii)		$y_{ij}=\mu+\alpha_i+e_{ij}$ μ = population … …grand mean for whole exp't α_i = population mean amount by which the *i*th treatment differs from μ	B2 B1 B1 B1	B1 if any two RHS terms correct "Population"; award here <u>or</u> for α_i e_{ij} is experimental error – does not need to be stated explicitly here, is subsumed in error assumptions below.
			e_{ij} ~ ind N **[*] (** accept "uncorrelated") (0 **[*]** , σ^2 **[*]**)	B2 **[7]**	if all three * components correct, B1 if any two correct.